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STRUCTURAL DAMAGE DETECTION BASED ON A MICRO-GENETIC ALGORITHM USING INCOMPLETE AND NOISY MODAL TEST DATA

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This paper describes a procedure for detecting structural damage based on a micro-genetic algorithm using incomplete and noisy modal test data. As the number of sensors used to measure modal data is normally small when compared with the degrees of freedom of the finite element model of the structure, the incomplete mode shape data are first expanded to match with all degrees of freedom of the finite element model under consideration. The elemental energy quotient difference is then employed to locate the damage domain approximately. Finally, a micro-genetic algorithm is used to quantify the damage extent by minimizing the errors between the measured data and numerical results. The process may be either of single-level or implemented through two-level search strategies. The study has covered the use of frequencies only and the combined use of both frequencies and mode shapes. The proposed method is applied to a single-span simply supported beam and a three-span continuous beam with multiple damage locations. In the study, the modal test data are simulated numerically using the finite element method. The measurement errors of modal data are simulated by superimposing random noise with appropriate magnitudes. The effectiveness of using frequencies and both frequencies and mode shapes as the data for quantification of damage extent are examined. The effects of incomplete and noisy modal test data on the accuracy of damage detection are also discussed.

1. INTRODUCTION

Civil engineering structures such as bridges and buildings are important assets to a nation. However, they may be damaged during their service lives due to various reasons.

Undetected damage may cause catastrophic failure leading to loss of lives and properties. It is certainly desirable to obtain information on the occurrence, the geometric location and the severity of the structural damage at the earliest possible stage. If the "state of health" of an important structure is constantly being monitored, its maintenance cost can be kept to a reasonable minimum.

Damage detection methods of structural systems based on changes in their vibration characteristics have been widely employed during the last two decades. Generally speaking, existing methods include those based on examination of changes in frequencies, mode shapes or mode shape curvatures, those based on dynamically measured flexibility and those based on updating structural model parameters, etc. Doebling *et al.* [1] published a state-of-the-art review on vibration-based damage identification methods developed up to 1998. Since then, research work in this field continued to flourish [2–10]. They include the sensitivity and statistical-based method by Messina *et al.* [2], the modal vibration characterization method using the vibratory residual forces and weighted sensitivity analysis by Kosmatka and Ricles [3], the frequency- and curvature- based experimental method by Ratcliffe [4], the method based on frequency measurement assuming concentrated damage by Vestroni and Capecchi [5], the detection method using singular value decomposition to deal with variations in dynamic behaviour by Ruotolo *et al.* [6], the method using modal and sensor norms by Gawronski and Sawicki [7], the measurement of operational deflection shapes by a scanning laser vibrometer by Pai and Jin [8], an assessment method using a quadratic programming model while minimizing the use of global numerical models and key material information by Hu *et al.* [9], a method for large-scale structures using super-elements with the concept of damage-detection-orientation-modelling [10], etc. Among various vibration-based damage detection methods described, those based on updating structural model parameters can be reduced to the solution of constrained optimization problems. Comparisons of the updated model parameters with the original correlated model parameters provide an indication of damage and can be used to quantify the location and extent of damage. However, for optimization problems in which the objective function has many local maxima and minima, or when the variables are combinations of many discrete and continuous variables, it is difficult to use conventional optimization algorithms such as the conjugate gradient method to obtain the global optimum.

In the last two decades, genetic algorithms [11] have been recognized as promising intelligent search techniques for difficult optimization problems. Genetic algorithms are stochastic search techniques based on the mechanism of natural selection and natural genetics. They combine the principle of survival of the fittest among string structures with a structured yet randomized information exchange to form a search algorithm with some of innovative flair of human search. Genetic algorithms have actually found their applications in structural damage identification [12–16].

Mares and Surace [12] employed a genetic algorithm to identify damage in elastic structures. In their study, a modified version of residual force vectors in terms of the stiffness matrix of the damaged structure was chosen as an objective function to be minimized while stiffness reduction factors of all elements were chosen to be variables. It implicitly means that the number of variables is equal to that of the elements in the finite element (FE) model and therefore the damage detection procedure proposed is time consuming for complex structures. In the study by Friswell *et al.* [13], a genetic algorithm was used to optimize the discrete damage location variables and for a given damage location site or sites, a standard eigensensitivity method was utilized to optimize the damage extent. The discrete variables represented the damage locations, which might cover all elements in the study. If a group of elements forming a possible damage domain can be first identified, the problem size will be reduced and consequently the efficiency and effectiveness of the final stage of damage quantification will be improved.

More recently, Xia and Hao [14] employed a genetic algorithm with real number encoding to identify the structural damage from vibration data. Three different criteria, namely the frequency changes, the mode shape changes and their combination, are considered in their study. Chou and Ghaboussi [15] used a genetic algorithm to solve an optimization problem formulated for detection and identification of structural damage. The "output error" indicating the difference between the measured and computed responses under static loading and the "equation error" indicating the residual force in the system of equilibrium equations are used to formulate the objective function to be optimized. Harrison and Butler [16] developed a two-stage procedure for locating delaminations within composite beams. To obtain the material properties and boundary conditions of the analytical model, they used a gradient-based optimization technique to minimize the difference of frequencies and mode shapes between the analytical model and the experimental data. Then they employed either a gradient-based optimization technique or a genetic algorithm to locate the delaminations. It was reported that a better solution was obtained by using a genetic algorithm.

In this paper, a procedure for detecting structural damage based on a micro-genetic algorithm using incomplete and noisy modal test data is described. In comparison with traditional genetic algorithms, a genetic algorithm with small populations is called micro-genetic algorithm. A serious limitation of traditional genetic algorithms is the time penalty involved in evaluating the fitness functions for large populations, particularly in complex problems [17]. Small populations could be used successfully with genetic algorithms if the population is restarted a sufficient number of times [18]. The mode shape data are incomplete because of the limited number of sensors used and the difficulty to measure the rotational degrees of freedom (d.o.f.s). These incomplete data are first expanded to match with all d.o.f.s of the FE model under consideration. The elemental energy quotient difference is then employed to locate the damage domain approximately. Finally a micro-genetic algorithm is used to quantify the damage extent by minimizing the errors between the measured data and numerical results. For a complex structure, the candidates of potential damage domain may still be many in spite of the use of elemental energy quotient difference. A two-level search strategy is therefore proposed for quantification of the damage extent, in which some of the potential damaged elements are first chosen and the damage extents are subsequently evaluated for these chosen elements. The efficiency of the proposed search strategy is compared with that of single-level search strategy that is usually employed in the literature.

To examine the effectiveness of using modal parameters for quantification of damage extent, two types of data have been used, i.e., the frequencies only and the combined use of both frequencies and mode shapes. The proposed method is applied to a single-span simply supported beam and a three-span continuous beam with multiple damage locations. The effects of incomplete and noisy modal test data on the accuracy of damage detection are also discussed.

2. DAMAGE DETECTION METHOD

2.1. EXPANSION OF INCOMPLETE MODE SHAPE DATA

As the number of sensors used to measure modal data is normally limited and the rotational d.o.f.s are difficult to measure, the resulting incomplete mode shape data have to be expanded to match with all d.o.f.s of the FE model under consideration. In the study, the system equivalent reduction expansion process (SEREP) [19] is employed to expand the incomplete mode shape data. In this method, the total modal matrix of a system is expressed as a linear combination of the eigenvectors from measured mode shapes at limited locations. The total modal matrices $\mathbf{\Phi}_{ua}$, $\mathbf{\Phi}_{um}$ and $\mathbf{\Phi}_{dm}$, and the mode shape matrices $\mathbf{\Phi}_{uaa}, \mathbf{\Phi}_{uma}$ and $\mathbf{\Phi}_{dma}$ at selected locations (i.e., the active d.o.f.s), respectively, are related by a transformation matrix $\mathbf{T}$ as follows:

$$\mathbf{\Phi}_{ua} = \begin{bmatrix} \mathbf{\Phi}_{uaa} \\ \mathbf{\Phi}_{uad} \end{bmatrix} = \mathbf{T}\mathbf{\Phi}_{uaa}, \tag{1}$$

$$\mathbf{\Phi}_{um} = \begin{bmatrix} \mathbf{\Phi}_{uma} \\ \mathbf{\Phi}_{umd} \end{bmatrix} = \mathbf{T}\mathbf{\Phi}_{uma}, \tag{2}$$

$$\mathbf{\Phi}_{dm} = \begin{bmatrix} \mathbf{\Phi}_{dma} \\ \mathbf{\Phi}_{dmd} \end{bmatrix} = \mathbf{T}\mathbf{\Phi}_{dma}, \tag{3}$$

where the first subscripts u and d denote the undamaged and damaged structures, respectively, the second subscripts a and m denote results from the FE analysis and the measured data, respectively, and the third subscripts a and d denote active and deleted d.o.f.s respectively. From equation (1), the transformation matrix $\mathbf{T}$ can be written as

$$\mathbf{T} = \mathbf{\Phi}_{ua}(\mathbf{\Phi}_{uaa}^{\mathrm{T}}\mathbf{\Phi}_{uaa})^{-1}\mathbf{\Phi}_{uaa}^{\mathrm{T}}. \tag{4}$$

Therefore, using the mode shapes obtained from the FE analysis, the transformation matrix $\mathbf{T}$ can be obtained and then used to expand the measured mode shapes at the active d.o.f.s to all d.o.f.s for both undamaged and damaged structures, respectively, according to equations (2) and (3). In the study, the measured modal test data are simulated numerically using the finite element method (FEM). The measurement errors of modal data are simulated by superimposing random noise with appropriate magnitudes.

2.2. LOCATION OF DAMAGE DOMAIN

Once the expanded mode shapes of the undamaged and damaged structures are available, the elemental energy quotient difference [20] is then employed to locate the damage domain approximately. The elemental energy quotient difference ($EEQD$) of the jth element corresponding to the ith mode is defined in terms of the elemental energy quotients $(EEQ)_{uij}$, and $(EEQ)_{dij}$, respectively, for the undamaged and damaged structures as

$$(EEQD)_{ij} = (EEQ)_{uij} - (EEQ)_{dij} = \frac{\mathbf{\Phi}_{umij}^{\mathrm{T}}\mathbf{K}_{uj}\mathbf{\Phi}_{umij}}{\mathbf{\Phi}_{umij}^{\mathrm{T}}\mathbf{M}_{j}\mathbf{\Phi}_{umij}} - \frac{\mathbf{\Phi}_{dmij}^{\mathrm{T}}\mathbf{K}_{uj}\mathbf{\Phi}_{dmij}}{\mathbf{\Phi}_{dmij}^{\mathrm{T}}\mathbf{M}_{j}\mathbf{\Phi}_{dmij}}, \tag{5}$$

where $\mathbf{K}_{uj}$ and $\mathbf{M}_j$ are the stiffness and mass matrices, respectively, of the jth element of the undamaged structure. The matrices $\mathbf{\Phi}_{umi}$ and $\mathbf{\Phi}_{dmi}$ contain the measured ith mode shapes of the undamaged and damaged structures respectively at the jth element. The quantity $(EEQD)_{ij}$ can be further normalized to become the elemental energy quotient difference ratio $(EEQDR)_{ij}$ as

$$(EEQDR)_{ij} = |(EEQD)_{ij}|/(EEQ)_{uij}. \tag{6}$$

If more than one measured mode is available, $(EEQDR)_{ij}$ is computed for all the modes and the parameter $(EEQDR)_j$ of the jth element is defined as the average of the quantities

$(EEQDR)_{ij}$ normalized with respect to the largest value $(EEQDR)_{i,\max}$ among all elements for each mode as

$$(EEQDR)_j = \frac{1}{m}\sum_{i=1}^{m}(EEQDR)_{ij}/(EEQDR)_{i,\max}, \tag{7}$$

where m is the number of measured mode shapes. In the process of damage detection, $(EEQDR)_j$ is computed for all the elements in the structure and those appearing as distinct local peaks might indicate the locations of damage. Such information is often useful in determining the possible damaged elements, which helps to limit the scope of search and reduce the subsequent computations necessary.

2.3. QUANTIFICATION OF DAMAGE EXTENT

When the structure has been damaged, the stiffness matrix $\mathbf{K}_{dj}$ of the damaged jth element can be represented as the stiffness matrix $\mathbf{K}_{uj}$ of the undamaged jth element multiplied by a stiffness reduction factor β_j reflecting the severity of the damage, i.e.,

$$\mathbf{K}_{dj} = \beta_j \mathbf{K}_{uj}, \tag{8}$$

Any possible non-linear behaviour of the damaged element is ignored for simplicity. The damage extent may then be quantified by minimizing the errors between the measured data and numerical results using a micro-genetic algorithm in which a small population size is utilized. The evaluation of errors may use either the frequencies only or a combination of both frequencies and mode shapes. The objective function F to be minimized can be written, respectively, as

$$F = \sum_{i=1}^{m}\left(\frac{\omega_{dmi} - \omega_{dai}}{\omega_{dmi}}\right)^2 \tag{9}$$

or

$$F = \sum_{i=1}^{m}\left(\frac{\omega_{dmi} - \omega_{dai}}{\omega_{dmi}}\right)^2 + \sum_{i=1}^{m}\frac{\|\mathbf{\Phi}_{dmi} - \mathbf{\Phi}_{dai}\|}{\|\mathbf{\Phi}_{dmi}\|}, \tag{10}$$

where ω_{dmi} and $\mathbf{\Phi}_{dmi}$ are, respectively, the measured frequency and mode shape of the ith mode of the damaged structure. Similarly, ω_{dai} and $\mathbf{\Phi}_{dai}$ are the numerical counterparts obtained from FE analysis. The variables to be optimized are the stiffness reduction factors β_j as defined in equation (8). The optimization of the stiffness reduction factors can be written as

$$\min_{\beta} F, \tag{11}$$

where the vector $\boldsymbol{\beta}$ contains the stiffness reduction factors β_j as

$$\beta = (\beta_1 \quad \beta_2 \quad \dots \quad \beta_{N_d})^{\mathrm{T}}, \tag{12}$$

where N_d is the number of possible damage elements in the damage domain. In this paper, the strategy to solve equation (11) directly is called the single-level search strategy. As a group of elements forming a possible damage domain have been identified using the parameter EEQDR as described above, the number of design variables is substantially reduced. However, the value of N_d may still be large for a complex structure and it may be much larger than its real value. For this case, a two-level search strategy is proposed in the paper to quantify the damage extent.

Let N_{rd} denote the assumed number of damaged elements in a structure, which should be much smaller than the value of N_d. In the two-level search strategy, N_{rd} elements are first chosen among the N_d candidates. Then the stiffness reduction factors of the N_{rd} chosen elements are to be optimized by minimizing the objective function as defined in equation (9) or (10). Another set of N_{rd} elements is then chosen and their reduction factors are optimized until an optimal set of N_{rd} elements is obtained. The damage extent is consequently evaluated in terms of the stiffness reduction factors corresponding to the optimal set of N_{rd} elements. The procedure described above can be expressed in mathematical form as follows:

$$\underset{I_d}{\text{Min}}\ F^*,$$

$$F^* = \underset{\beta_{I_d}}{\text{Min}}\ F, \tag{13}$$

where I_d is a vector whose components are the numberings of the elements chosen. It implies that the design variables in the first level optimization are discrete ones. The components of vector β_{I_d} are the stiffness reduction factors of the N_{rd} elements chosen and they are continuous design variables.

3. ILLUSTRATIVE EXAMPLES

To evaluate the efficiency and effectiveness of the proposed method, numerical simulations are carried out using modal test data, which may be complete or incomplete, and noise-free or noisy. A single-span simply supported beam and a three-span continuous beam are chosen for the purpose. In the study, the modal test data are simulated numerically using an FE model. Only the first 5 modes are utilized in damage detection. The natural frequencies are perturbed randomly by 1 and 2%, respectively, to simulate the measurement errors of modal data. Similarly, 5 and 10% random noise are superimposed on the exact mode shapes to obtain the noisy data. Complete modal test data and incomplete sets comprising only the translational d.o.f.s are separately tested. Six primary schemes as listed in Table 1 are considered for the single-span simply supported beam, while only Schemes 1, 4A (i.e., Sub-case A of Scheme 4) and 5 are considered for the three-span continuous beam. In essence, Schemes 1 and 5 use frequency data only, whereas the rest use both frequency and mode shape data. Schemes 3, 4 and 6 have Sub-cases A and B for noise level in mode shape data of 5 and 10%, respectively, as shown in Table 1. The stiffness reduction factors of possible damaged elements indicating damage extent are solved by using single-level and two-level search strategies. In the study, a micro-genetic algorithm with uniform crossover is employed using the following parameters: crossover rate $C = 0{\cdot}5$, jump mutation rate $M_j = 0{\cdot}02$, creep mutation rate $M_c = 0{\cdot}04$ and number of chromosomes (binary bits) per parameter $N_c = 10$. Both niching and elitism strategies are adopted in the genetic search procedure.

3.1. EXAMPLE 1: A SINGLE-SPAN SIMPLY SUPPORTED BEAM

A single-span simply supported beam is first considered. The characteristics of the original beam are as follows: modulus of elasticity $E = 3{\cdot}0 \times 10^{10}\,\text{N/m}^2$, mass density $\rho = 2400\,\text{kg/m}^3$, cross-sectional area $A = 1{\cdot}796\,\text{m}^2$, second moment of area of cross-section $I = 0{\cdot}2356\,\text{m}^4$ and span length $L = 20\,\text{m}$. In the study, the first five frequencies and mode shapes are simulated numerically using an FE model with 16 elements of equal length numbering sequentially from the left end to the right.

Table 1

Schemes for quantification of damage extent investigated

Scheme	Description	Percentage noise	
		Frequency (%)	Mode shape (%)
1	Frequencies with 1% noise only	1	—
2	Frequencies with 1% noise, and complete exact mode shapes	1	0
3A	Frequencies with 1% noise, and complete but noisy (5%) mode shapes	1	5
3B	Frequencies with 1% noise, and complete but noisy (10%) mode shapes	1	10
4A	Frequencies with 1% noise, and incomplete noisy (5%) mode shapes	1	5
4B	Frequencies with 1% noise, and incomplete noisy (10%) mode shapes	1	10
5	Frequencies with 2% noise only	2	—
6A	Frequencies with 2% noise, and incomplete noisy (5%) mode shapes	2	5
6B	Frequencies with 2% noise, and incomplete noisy (10%) mode shapes	2	10

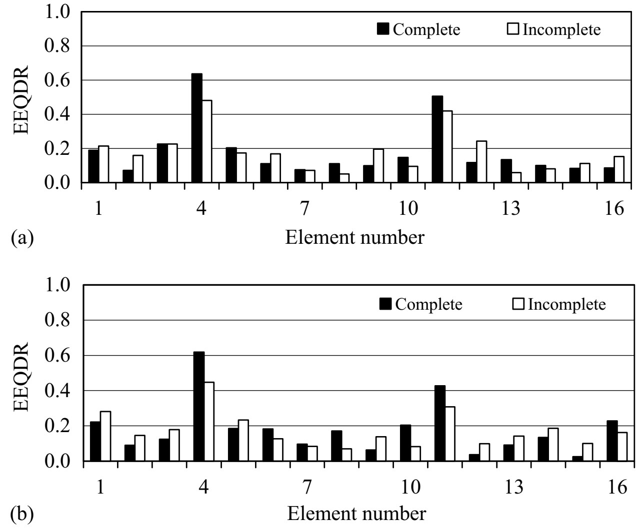

Figure 1. Single-span simply supported beam with damage at Elements 4 and 11: effect of incompleteness of mode shape data on damage detection. (a) 5% noise; (b) 10% noise.

Elements 4 and 11 have been damaged and this is simulated by reduction of their bending stiffnesses with the factors $\beta_4 = 0{\cdot}5$ and $\beta_{11} = 0{\cdot}5$. The damage locations identified by EEQDR using 5 and 10% noise in the mode shapes, with complete and incomplete modal test data, are shown in Figures 1 and 2. In particular, Figure 1 focuses on the effects of incompleteness while Figure 2 focuses on the effects of noise. It can be seen that the use of the incomplete modes, i.e., all translational d.o.f.s only, can still locate the damage domain approximately although it does have some adverse effects on the identification results. It is also noted that relatively high noise in the mode shapes has adverse effect on the accuracy of damage location, particularly if incomplete mode shapes are used. Generally speaking, the parameter EEQDR can be employed to approximately locate the damage spots of the structure under consideration using the incomplete mode shapes with 5 and 10% random noise. Six different cases of possible damage domains are assumed to examine their effect on the accuracy of damage extent determined, as shown in Table 2. Notice that in Case 6, all 16 elements have been included in the possible damage domain for comparison purpose. The stiffness reduction factors of possible damage elements are computed using the proposed method with single-level search strategy and given in Table 3.

Comparing the results of Schemes 1 and 5 in which only frequencies are used, it is found that high noise in natural frequencies generally has adverse effects on the accuracy of evaluating damage extent. It is also noted that incomplete modal test data decrease the accuracy of damage detection, as is evident from examining the results for Schemes 3A, 3B, 4A and 4B. The accuracy of damage extents tends to decrease as the number of

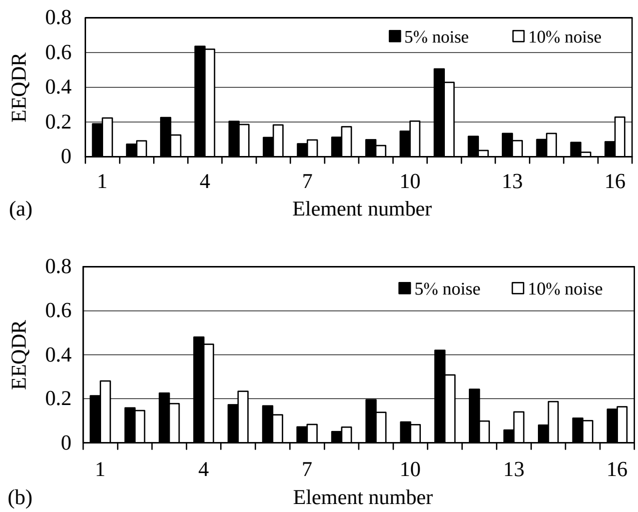

Figure 2. Single-span simply supported beam with damage at Elements 4 and 11: effect of noise of mode shape data on damage detection. (a) Complete modal data; (b) incomplete modal data.

TABLE 2

Assumed damage domain for the single-span simply supported beam

Case	Damage domain
1	Elements 4 and 11
2	Elements 4, 11 and 14
3	Elements 4, 11 and 16
4	Elements 1, 4, 11 and 14
5	Elements 1, 4, 11 and 16
6	All 16 elements in the finite element model

TABLE 3

Evaluated damage extent for the single-span simply supported beam

Damage domain	Scheme 1	Scheme 2	Scheme 3A	Scheme 3B	Scheme 4A	Scheme 4B	Scheme 5	Scheme 6A	Scheme 6B
Case 1									
4	*0.4658*	*0.5007*	*0.5313*	*0.4668*	*0.5254*	*0.5664*	*0.4219*	*0.5756*	*0.5721*
11	*0.4541*	*0.4932*	*0.4766*	*0.5029*	*0.5645*	*0.5381*	*0.4287*	*0.5585*	*0.5371*
Case 2									
4	*0.4678*	*0.4902*	*0.4629*	*0.4570*	*0.4932*	*0.5371*	*0.5078*	*0.4736*	*0.5527*
11	*0.5078*	*0.5010*	*0.5079*	*0.5039*	*0.5791*	*0.5801*	*0.4326*	*0.5539*	*0.5840*
14	0.9760	0.9922	0.9375	0.9121	0.9453	0.8941	0.8047	0.9207	0.8841
Case 3									
4	*0.4492*	*0.5039*	*0.4727*	*0.4287*	*0.5513*	*0.5275*	*0.4463*	*0.5147*	*0.5335*
11	*0.5000*	*0.5039*	*0.5156*	*0.4668*	*0.5547*	*0.5635*	*0.4482*	*0.5586*	*0.5804*
16	0.9289	0.9668	0.8721	0.8295	0.8613	0.8193	0.8019	0.8154	0.8030
Case 4									
1	0.9666	0.9844	0.9922	0.9736	0.9951	0.9775	0.8145	0.9854	0.9767
4	*0.4600*	*0.4941*	*0.4679*	*0.4434*	*0.5449*	*0.5645*	*0.4717*	*0.5613*	*0.6250*
11	*0.4785*	*0.5020*	*0.5073*	*0.4766*	*0.5674*	*0.5527*	*0.4355*	*0.5731*	*0.6250*
14	0.9678	0.9824	0.9297	0.9021	0.9795	0.8736	0.8467	0.9049	0.9453
Case 5									
1	0.9164	0.9922	0.9990	0.9326	0.9766	0.9980	0.8443	0.9805	0.9643
4	*0.4521*	*0.4990*	*0.5000*	*0.4434*	*0.5156*	*0.5449*	*0.4258*	*0.5156*	*0.5664*
11	*0.5000*	*0.5156*	*0.4766*	*0.5010*	*0.5879*	*0.5361*	*0.4688*	*0.5723*	*0.5406*
16	0.9568	0.9902	0.8506	0.8666	0.8262	0.8887	0.8863	0.8242	0.8651
Case 6									
1	0.9971	0.9531	0.9941	0.9990	0.9990	0.9980	0.9775	0.9500	0.9790
2	0.9890	0.9678	0.9790	0.9781	0.8750	0.7646	0.9688	0.8541	0.7326
3	0.9762	0.9512	0.9668	0.9648	0.9990	0.9980	0.9727	0.8924	0.9421
4	*0.5420*	*0.4873*	*0.4512*	*0.4053*	*0.5146*	*0.4951*	*0.5313*	*0.5049*	*0.4854*
5	0.8877	0.9678	0.9697	0.9990	0.9541	0.9491	0.7646	0.9755	0.9660
6	0.7250	0.9639	0.9509	0.9951	0.9990	0.9540	0.6406	0.9990	0.9653
7	0.9961	0.9717	0.9473	0.9404	0.9385	0.9639	0.9980	0.8916	0.9473
8	0.9980	0.9365	0.8906	0.8271	0.9336	0.9385	0.9990	0.9297	0.9188
9	0.9990	0.9580	0.9814	0.9491	0.9150	0.7490	0.9990	0.9072	0.7200
10	0.9102	0.9688	0.8926	0.8691	0.8008	0.7549	0.8115	0.7490	0.7172
11	*0.6500*	*0.4854*	*0.4984*	*0.4980*	*0.5469*	*0.5342*	*0.7617*	*0.5479*	*0.5225*
12	0.9199	0.9757	0.9932	0.9912	0.8145	0.7412	0.9492	0.8750	0.7520
13	0.8623	0.9688	0.9970	0.9620	0.9851	0.9766	0.8506	0.9990	0.9512
14	0.9702	0.9600	0.8975	0.8486	0.9766	0.9863	0.9531	0.9609	0.9971
15	0.9491	0.9600	0.9736	0.9668	0.9688	0.8779	0.9854	0.9532	0.8477
16	0.9961	0.9609	0.8789	0.7852	0.8105	0.8545	0.9970	0.7959	0.8936

Note: figures in italics correspond to the damaged elements.

elements forming a possible damage domain increases, which implies that incompleteness of mode shapes and higher noise in mode shapes have adverse effects on the accuracy of damage detection. This is because the use of incomplete and noisy mode shapes results in a larger candidate set of damaged elements and this phenomenon is clearly shown in the EEQDR plot in Figure 2(b). The use of polluted mode shapes together with the frequencies may have favourable or unfavourable effects on the accuracy of damage detection. For example, using the noisy mode shapes and measured frequencies with 1%

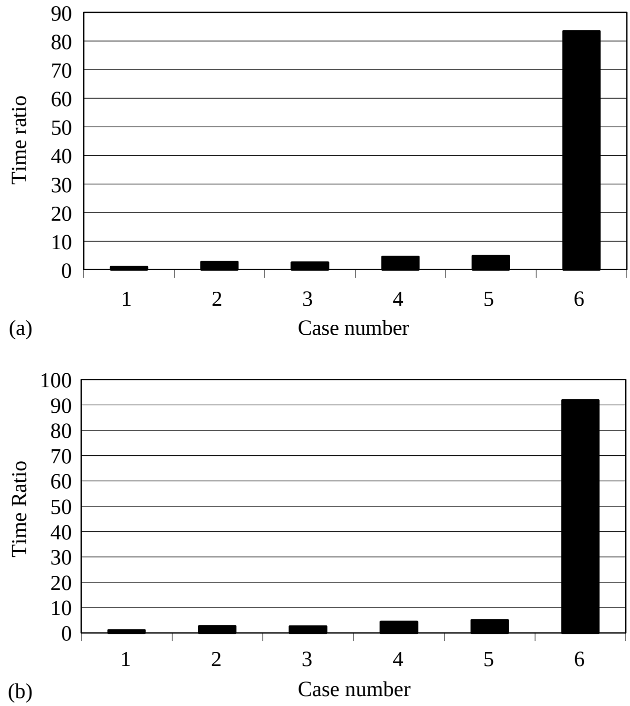

Figure 3. Time ratios for Cases 1–6 for the single-span simply supported beam with damage at Elements 4 and 11. (a) Scheme 1; (b) Scheme 2.

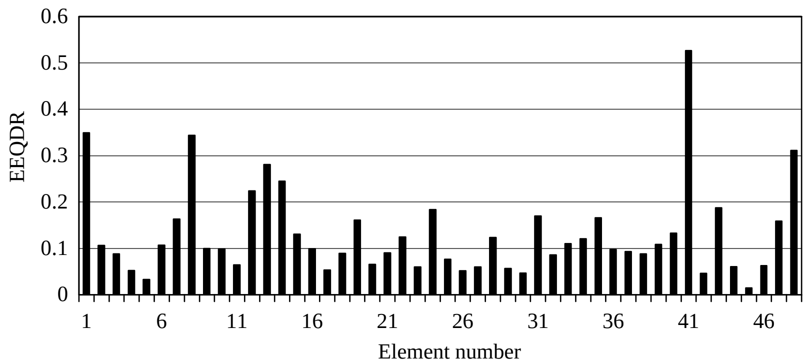

Figure 4. Three-span continuous beam with damage at Elements 8 and 41: EEQDR plot using incomplete mode shapes with 5% noise.

noise worsens the results for the structure considered. However, using the noisy mode shapes together with 2% noise in frequencies improves them. When the exact mode shapes are used together with frequencies with 1% noise, the damage extents of Elements 4 and 11, which have been assumed damaged, are very accurately evaluated.

For each of the six different damage domains listed in Table 2, the total computation time is recorded, and it is indicative of the computational effort to converge to the stiffness reduction factors. Taking the time of Case 1 as the benchmark, the time ratio for different damage domains can be computed, which are shown in Figures 3(a) and 3(b) for Schemes

TABLE 4

Assumed damage domain for the three-span continuous beam

Case	Damage domain	Assumed no. of damaged elements	Search strategy
1	1, 8, 13, 19, 22, 24, 28, 31, 35, 41, 43, 48	12	Single-level
2	1, 8, 13, 19, 22, 24, 28, 31, 35, 41, 43, 48	2	Two-level
3	1, 8, 13, 19, 22, 24, 28, 31, 35, 41, 43, 48	3	Two-level
4	1, 8, 13, 19, 22, 24, 28, 31, 35, 41, 43, 48	4	Two-level

TABLE 5

Evaluated damage extent for the three-span continuous beam

Case	Assumed no. of damaged elements	Scheme 1		Scheme 4A		Scheme 5	
		Elements identified	Damage extent	Elements identified	Damage extent	Elements identified	Damage extent
1	12	1	0.9678	1	0.9756	1	0.9521
		8	*0.5059*	*8*	*0.5322*	*8*	*0.5010*
		13	0.9277	13	0.9365	13	0.9980
		19	0.9629	19	0.9971	19	0.9375
		22	0.9795	22	0.8760	22	0.8340
		24	0.9785	24	0.9355	24	0.9658
		28	0.9766	28	0.9932	28	0.8496
		31	0.9492	31	0.8184	31	0.9873
		35	0.9980	35	0.9805	35	0.7793
		41	*0.5000*	*41*	*0.4990*	*41*	*0.5430*
		43	0.9375	43	0.9873	43	0.9902
		48	0.9414	48	0.9473	48	0.9814
2	2	*8*	*0.4845*	*8*	*0.5391*	*8*	*0.4766*
		41	*0.4463*	*41*	*0.5547*	*41*	*0.4084*
3	3	8	0.4375	8	0.4844	8	0.4766
		22	0.9375	35	0.9297	*41*	*0.5703*
		41	*0.5313*	*41*	*0.5073*	48	0.8125
4	4	*8*	*0.5234*	*8*	*0.4453*	*8*	*0.5000*
		22	0.9219	*41*	*0.4765*	22	0.9422
		28	0.9844	43	0.9140	*41*	*0.4063*
		41	*0.4297*	48	0.9063	48	0.8823

Note: figures in italics correspond to the damaged elements.

1 and 2 respectively. It is observed that the time ratio greatly increases as the number of elements in the damage domain increases. This is because the search space expands considerably as the number of elements included in the damage domain increases, and therefore the time to search for the optimal solution increases.

3.2. EXAMPLE 2: A THREE-SPAN CONTINUOUS BEAM

A three-span continuous beam is next considered. Each span is equal to 20 m and all other parameters of the beam are the same as in Example 1. The modal test data for the study are simulated using an FE model with 48 elements of equal length numbering sequentially from the extreme left end to the right. Elements 8 and 41 have been damaged and their stiffness reduction factors are $\beta_8 = 0{\cdot}5$ and $\beta_{41} = 0{\cdot}5$. The plot of EEQDR using incomplete modal test data comprising only the translational d.o.f.s with 5% noise are depicted in Figure 4. From the figure, Elements 1, 8, 13, 19, 22, 24, 28, 31, 35, 41, 43 and 48

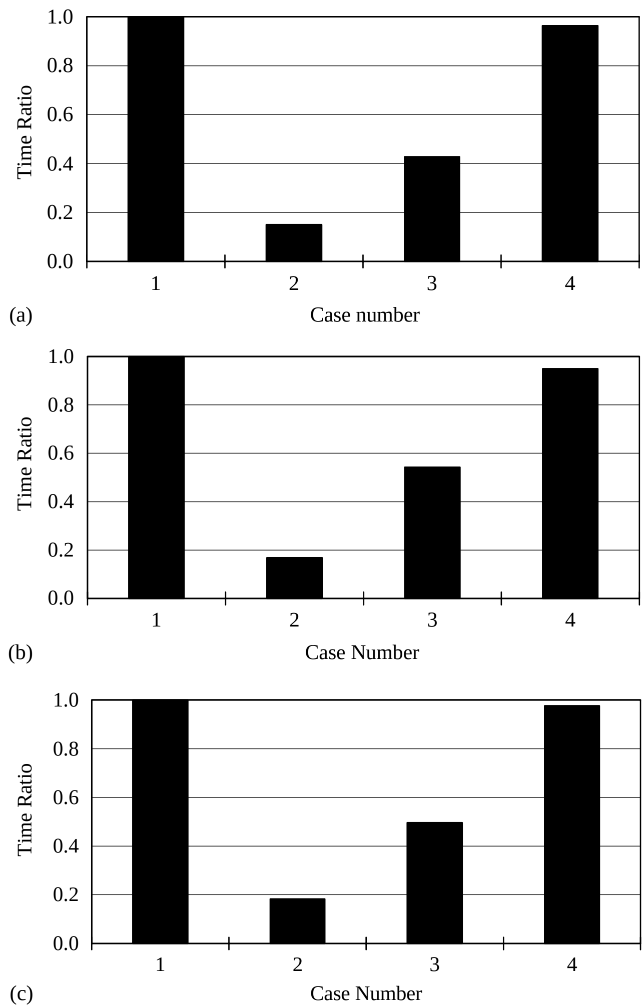

Figure 5. Time ratios for Cases 1–4 for the three-span continuous beam with damage at Elements 8 and 41 (a) Scheme 1; (b) Scheme 4A; (c) Scheme 5.

are identified as possible damage elements. To evaluate the efficiency of the two search strategies as described above, the four cases as listed in Table 4 are tested for Schemes 1, 4A and 5 as described in Table 1. The evaluated damage extents for Cases 1–4 are shown in Table 5. The effects of noise in frequency and mode shape data have similar trends as in Example 1.

Time ratios are also computed for different cases with respect to that of Case 1, which is taken as the benchmark. Figures 5(a), 5(b) and 5(c) show the time ratio for Schemes 1, 4A and 5 respectively. It can be observed from Figure 5 that the two-level search strategy saves computation time while it is still capable of obtaining satisfactory results of evaluated damage extent for the beam under consideration.

4. CONCLUSIONS

A procedure for detecting structural damage using noisy and incomplete modal data is described in this paper. The incomplete mode shape data are first expanded using the system equivalent reduction expansion process to match with all degrees of freedom of the finite element model of the structure under consideration. The element energy quotient difference is then employed to locate the damage domain approximately. Finally, a micro-genetic algorithm is used to quantify the damage extent by minimizing the errors between the measured data and numerical predictions with different search strategies. Numerical results show that incomplete and noisy modal data have adverse effect on the accuracy of damage detection. However, the use of polluted mode shapes together with the frequencies may have favourable or unfavourable effects on the accuracy of damage detection. In addition, the two-level search strategy proposed is often more efficient in quantification of damage extent than the single-level strategy when there are a lot of candidates in the possible damage domain.

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